

## **Hierarchy of Symmetry Breakings and Neutral Currents in an $SU(6)$ Grand Unification Model**

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In an  $SU(6)$  grand unification model with eight quarks and eight leptons belonging to 15-plet and singlet representations, the symmetry is spontaneously broken by the sequence  $SU(6) \rightarrow SU(3)^c \times SU(2) \times U(1) \times U(1) \rightarrow SU(3)^c \times U(1)$ . For two cases of symmetry breakings the effective weak neutral current coupling constants are compared with experiment. For the  $SU(3)^c \times SU(2) \times U(1) \times U(1) \rightarrow SU(3)^c \times U(1)$  symmetry breaking, the coupling constants reproduce the Weinberg-Salam model with a small correction term. Agreement with the experimental mean values is improved with the correction term. Parity violation in atomic physics is also discussed.

The standard  $SU(2) \times U(1)$  gauge theory (Weinberg, 1967; Salam, 1968) of weak and electromagnetic interactions has been successful in predicting the existence of the charmed quark (Aubert et al., 1974; Augustin et al., 1974; Bacci et al., 1974) in connection with GIM (Glashow-Iliopoulos-Maiani) mechanism (Glashow, 1970). With the observation (Hasert, 1973; Benvenuti, 1974; Blietschau, 1977; Benvenuti et al., 1976; Holder, 1977) of the predicted weak neutral current, the confidence in the extended  $SU(2) \times U(1)$  model with four quarks and leptons was further raised. The recently observed upsilon (9.4 GeV) (Herb et al., 1977) which is interpreted as a bound state of  $b$  and  $\bar{b}$  quarks, and the heavy lepton  $\tau$  (1.9 GeV) (Perl et al., 1975, 1976) necessitate the further extension of the model. The  $SU(3) \times U(1)$  models seem to be ruled out by the analysis (Abbot and Barnett, 1978) of the weak neutral current cou-

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pling constants which favorably supports the Weinberg–Salam (WS) model (Weinberg, 1967; Salam, 1968). The WS model still has one unresolved problem with its predicted parity violation in atomic physics. The experiments in this area are still so uncertain that any conclusive settlement has to wait further experimental result.

Under these circumstances, it would be meaningful to unify further with strong interaction in terms of the quantum chromodynamics (Fritzsch et al. 1972; Gross and Wilczek, 1973; Weinberg, 1973) (QCD) with the exact color  $SU(3)^c$  symmetry and an octet of colored neutral massless gluons such that the WS model is embedded in it. There are attempts to enlarge the symmetry group  $G$  such that  $G$  contains  $SU(3)^c \times SU(2) \times U(1) \times \dots$ . In the grand unification models such as  $SU(5)$  (Georgi and Glashow, 1974; Buras et al., 1978) and  $SU(6)$  (Inoue et al., 1977a, b<sup>2</sup>) models, the symmetry group imposes mass relations among the leptons and quarks after the symmetry is broken spontaneously by the Higgs mechanism. The resulting mass relations among fermions and the mixings of their states provide additional constraints on the models. The  $SU(5)$  model gives mass relations (Georgi and Glashow, 1974) like  $m_e = m_\mu$  and  $m_\mu = m_s$ , etc., after the spontaneous breaking of the symmetry down to  $SU(3)^c \times U(1)$ . If the electromagnetic and gluon radiative corrections are included, this may further split the fermion masses.<sup>3</sup> There are no other useful mass relations in the  $SU(5)$  model and no explanation of the Cabibbo angle in this model.

As for the  $SU(6)$  model discussed in Yun (1978a, b), we have mass relations such as  $m(d'_i) + m(s'_i) + m(b'_i) + m(h'_i) = m(e^{-'}) + m(\mu^{-'}) + m(E^{-'}) + m(M^{-'})$  and  $m(d'_i)/m(b'_i) = m(s'_i)/m(h'_i) = |b/a|$  and it predicts the masses of  $b'_i$  and  $h'_i$  quarks, respectively,  $m(b'_i) = 3.1 \sim 4.8$  GeV and  $m(h'_i) = 4.1 \sim 6.2$  GeV, where the primes indicate the diagonalized states and the color indices  $i = 1, 2, 3$ . This prediction was possible after generating the Cabibbo angle by the diagonalization of mass matrix.

In this paper, we consider a hierarchy of symmetry breakings in the sequence,  $SU(6) \rightarrow SU(3)^c \times SU(2) \times U(1) \times U(1) \rightarrow SU(3)^c \times U(1)$ , and obtain the effective weak neutral current coupling constants for  $SU(6) \rightarrow SU(3)^c \times U(1)$  and  $SU(3)^c \times SU(2) \times U(1) \times U(1) \rightarrow SU(3)^c \times U(1)$  breakings. Comparing them with the experimental results in the second case, the relative mass ratios of the neutral vector bosons  $Z$  and  $Y$  to  $W$  are obtained at the level of  $SU(3)^c \times U(1)$ . The neutral current coupling constants in the second case of symmetry breaking are exactly the ones obtained by the WS model in the limit of  $m(Z)/m(Y) \rightarrow \text{large}$ . The correction terms to the coupling constants are related to this mass ratio.

<sup>2</sup>These works, as indicated in Yun (1978a, b) have similar features but quite different symmetry-breaking mechanisms from the present work. They have no discussions on the fermion masses and their diagonalized states.

<sup>3</sup>In Buras et al. (1978) the mass corrections due to the renormalization effect are obtained.

We consider the following two left-handed (right-handed) antisymmetric 15-plets and two left-handed (right-handed) singlets of fermions:

$$\begin{aligned}
 L_1 &= \frac{1}{2^{1/2}} \left[ \begin{array}{ccc|ccc} 0 & \bar{u}_3 & -\bar{u}_2 & -u_1 & -d_1 & -b_1 \\ -\bar{u}_3 & 0 & \bar{u}_1 & -u_2 & -d_2 & -b_2 \\ \bar{u}_2 & -\bar{u}_1 & 0 & -u_3 & -d_3 & -b_3 \\ \hline u_1 & u_2 & u_3 & 0 & e^+ & -E^+ \\ d_1 & d_2 & d_3 & -e^+ & 0 & \bar{E}^0 \\ b_1 & b_2 & b_3 & E^+ & -\bar{E}^0 & 0 \end{array} \right]_L, \\
 R_1 &= \frac{1}{2^{1/2}} \left[ \begin{array}{ccc|ccc} 0 & \bar{t}_3 & -\bar{t}_2 & -t_1 & -b_1 & -d_1 \\ -\bar{t}_3 & 0 & \bar{t}_1 & -t_2 & -b_2 & -d_2 \\ \bar{t}_2 & -\bar{t}_1 & 0 & -t_3 & -b_3 & -d_3 \\ \hline t_1 & t_2 & t_3 & 0 & E^+ & -e^+ \\ b_1 & b_2 & b_3 & -E^+ & 0 & \bar{\nu}_e \\ d_1 & d_2 & d_3 & e^+ & -\bar{\nu}_e & 0 \end{array} \right]_R, \\
 L_2 &= \frac{1}{2^{1/2}} \left[ \begin{array}{ccc|ccc} 0 & \bar{c}_3 & -\bar{c}_2 & -c_1 & -s_1 & -h_1 \\ -\bar{c}_3 & 0 & \bar{c}_1 & -c_2 & -s_2 & -h_2 \\ \bar{c}_2 & -\bar{c}_1 & 0 & -c_3 & -s_3 & -h_3 \\ \hline c_1 & c_2 & c_3 & 0 & \mu^+ & -M^+ \\ s_1 & s_2 & s_3 & -\mu^+ & 0 & M^0 \\ h_1 & h_2 & h_3 & -M^+ & \bar{M}^0 & 0 \end{array} \right]_L, \\
 R_2 &= \frac{1}{2^{1/2}} \left[ \begin{array}{ccc|ccc} 0 & \bar{g}_3 & -\bar{g}_2 & -g_1 & -h_1 & -s_1 \\ -\bar{g}_3 & 0 & \bar{g}_1 & -g_2 & -h_2 & -s_2 \\ \bar{g}_2 & -\bar{g}_1 & 0 & -g_3 & -h_3 & -s_3 \\ \hline g_1 & g_2 & g_3 & 0 & M^+ & -\mu^+ \\ h_1 & h_2 & h_3 & -M^+ & 0 & \bar{\nu}_\mu \\ s_1 & s_2 & s_3 & \mu^+ & -\bar{\nu}_\mu & 0 \end{array} \right]_R,
 \end{aligned} \tag{1}$$

$$L_3 = (\bar{\nu}_e)_L, \quad R_3 = (\bar{E}^0)_R$$

$$L_4 = (\bar{\nu}_\mu)_L, \quad R_4 = (\bar{M}^0)_R$$

The  $SU(6)$  invariant Lagrangian is

$$\mathcal{L} = \frac{1}{2} ig_6 \sum_i \text{Tr}(\bar{L}_i \gamma_\mu \lambda^\alpha L_i + \bar{R}_i \gamma_\mu \lambda^\alpha R_i) V_\alpha^\mu \quad (2)$$

where  $\alpha = 1, \dots, 35$ .

The charge operator  $\mathcal{Q}$  is

$$\mathcal{Q} = I_3 + \frac{Y}{2} + Y = \frac{1}{2} \left( \lambda_3 + \frac{1}{3^{1/2}} \lambda_8 + \frac{4}{3^{1/2}} \lambda_0 \right) \quad (3)$$

where

$$\lambda_3 = \begin{bmatrix} 0 & & & & & \\ & 0 & & & & \\ & & 0 & & & \\ \hline & & & 1 & & \\ & & & & -1 & \\ & 0 & & & & 0 \end{bmatrix},$$

$$\lambda_8 = \frac{1}{3^{1/2}} \begin{bmatrix} 0 & & & & & \\ & 0 & & & & \\ & & 0 & & & \\ \hline & & & 1 & & \\ & & & & 1 & \\ & 0 & & & & -2 \end{bmatrix},$$

$$\lambda_0 = \frac{1}{3^{1/2}} \begin{bmatrix} -1 & & & & & \\ & -1 & & & & \\ & & -1 & & & \\ \hline & & & -1 & & \\ & & & & 1 & \\ & & 0 & & & 1 \\ & & & & & & 1 \end{bmatrix}$$

This is due to the charge structure of fermions which is shown in Table I. The  $6 \times 6$  matrix representation of the 35 vector bosons  $V_j^i$ , where  $i, j = 1, \dots, 6$ , has the following diagonal elements:

$$\begin{aligned} V_1^1 &= V_2^2 = V_3^3 = -\frac{1}{3^{1/2}} V_0 \\ V_4^4 &= V_3 + \frac{1}{3^{1/2}} V_8 + \frac{1}{3^{1/2}} V_0 \\ V_5^5 &= -V_3 + \frac{1}{3^{1/2}} V_8 + \frac{1}{3^{1/2}} V_0 \\ V_6^6 &= -\frac{2}{3^{1/2}} V_8 + \frac{1}{3^{1/2}} V_0 \end{aligned} \quad (4)$$





TABLE II. Comparison of weak neutral current coupling constants in the case of the symmetry-breaking  $SU(3)^c \times SU(2) \times U(1) \times U(1) \rightarrow SU(3)^c \times U(1)$ .

	The present model [equation (16)]	WS model	Experiment <sup>a</sup>
$C_{u_L} = C_{c_L}$	0.377	0.353	$0.33 \pm 0.07$
$C_{d_L} = C_{s_L}$	-0.403	-0.427	$-0.40 \pm 0.07$
$C_{u_R} = C_{c_R}$	-0.195	-0.147	$-0.18 \pm 0.06$
$C_{d_R} = C_{s_R}$	0.025	0.073	$0 \pm 0.11$

<sup>a</sup>From Abbot and Barnett (1978).

where  $A \equiv [m^2(w)/2m^2(Z)]$ , and there is no simple relation between  $m(w)$  and  $m(Z)$  in this case (Yun, 1978a, b). Note that  $Y$  does not contribute here. By comparing with experimental values given in Table II we see that the coupling constants for  $u_R$  and  $d_R$  are not as good as the ones for  $u_L$  and  $d_L$  for the value of  $A \simeq 1$ . The symmetry breaking of  $SU(6) \rightarrow SU(3)^c \times U(1)$  gives poor values for right-handed couplings. The neutral current couplings of electrons in this case is such that  $\bar{e}e$  is pure vector, and  $\bar{u}u$  and  $\bar{d}d$  are pure axial vectors as far as their couplings through  $Z$  are concerned. However, through  $Y$ ,  $\bar{e}e$  and  $\bar{d}d$  are pure axial vectors with vanishing  $\bar{u}u$ . Overall, parity violation in atomic physics is suppressed in this case of symmetry breaking.

At the level of  $SU(3)^c \times SU(2) \times U(1) \times U(1)$  symmetry, the 15-plet fermion states are broken down to  $SU(2)$  doublets and singlets, as follows:

$$\begin{array}{cccc}
 \begin{pmatrix} u_i \\ d_i \end{pmatrix}_L & \begin{pmatrix} t_i \\ b_i \end{pmatrix}_R & \begin{pmatrix} c_i \\ s_i \end{pmatrix}_L & \begin{pmatrix} g_i \\ h_i \end{pmatrix}_R \\
 b_{iL} & d_{iR} & h_{iL} & s_{iR} \\
 \bar{u}_{iL} & \bar{t}_{iR} & \bar{c}_{iL} & \bar{g}_{iR} \\
 \begin{pmatrix} E^+ \\ \bar{E}^0 \end{pmatrix}_L & \begin{pmatrix} e^+ \\ \bar{\nu}_e \end{pmatrix}_R & \begin{pmatrix} M^+ \\ \bar{M}^0 \end{pmatrix}_L & \begin{pmatrix} \mu^+ \\ \bar{\nu}_\mu \end{pmatrix}_R \\
 e_L^+ & E_R^+ & \mu_L^+ & M_R^+ \\
 \bar{\nu}_{eL} & \bar{E}_R^0 & \bar{\nu}_{\mu L} & M_R^0
 \end{array} \tag{8}$$

Their quantum numbers are shown in Table I. The  $SU(2) \times U(1) \times U(1)$  invariant weak interaction is given by

$$\mathcal{L}_w = \left(\frac{1}{2}i\right) \left[ g\bar{\psi}\gamma_\mu\tau\psi A^\mu + g'\bar{y}\bar{\psi}\gamma_\mu\psi B^\mu + g''\bar{\psi}\gamma_\mu 2Y\psi C^\mu \right] \tag{9}$$

where  $\psi$  is the  $SU(2)$  doublets or singlets given in (8). It is noted here that  $(d_i, s_i, b_i, h_i)$  as well as leptons are already mixed at this stage as shown in

Yun (1978a, b). Thus heavy quarks such as  $b_i$  decay to light quarks via these mixed states. Due to the undetermined mixing angles, there is not much we can say at this moment. If we regard  $E^+$  as a  $\tau^+$  heavy lepton, this model may have predominantly right-handed coupling even after mixing, which may be a problem. One way to modify this is to add another 15-plet which contains  $\tau^+$  and  $\nu_\tau$  in the right-handed state.

We now introduce the Higgs mechanism by an  $SU(2)$  doublet and a singlet,

$$\begin{array}{r} \langle \phi_1 \rangle = \begin{pmatrix} 0 \\ \lambda_1 \end{pmatrix} \\ \langle \phi_2 \rangle = (\lambda_2) \end{array} \begin{array}{ccc} I_3 & y & Y \text{ } \mathcal{Q} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{3} \text{ } +1 \\ -\frac{1}{2} & \frac{1}{3} & \frac{1}{3} \text{ } 0 \\ 0 & \frac{2}{3} & -\frac{1}{3} \text{ } 0 \end{array} \quad (10)$$

The neutral vector bosons are mixed as

$$\begin{aligned} A_3 &= (Z \cos \phi - A \sin \phi) \\ B &= [(Z \sin \phi + A \cos \phi) \cos \alpha - Y \sin \alpha] \\ C &= [(Z \sin \phi + A \cos \phi) \sin \alpha + Y \cos \alpha] \end{aligned} \quad (11)$$

The charge structure of fermions imposes that

$$\begin{aligned} \frac{1}{e^2} &= \frac{1}{g^2} + \frac{1}{g'^2} + \frac{1}{g''^2} \\ e &= -g \sin \phi = g' \cos \phi \cos \alpha = g'' \cos \phi \sin \alpha \end{aligned} \quad (12)$$

and the diagonalization of mass matrix for the neutral vector bosons  $Z$  and  $Y$  demands

$$\cos^2 \alpha = \frac{1}{3} \quad (13)$$

The masses of the vector bosons are acquired by

$$\begin{aligned} \mathcal{L}_\phi &= \left| \left( \partial_\mu - i \frac{g}{2} \boldsymbol{\tau} \cdot \mathbf{A}_\mu - i \frac{g'}{2} y B_\mu - i \frac{g''}{2} 2 Y C_\mu \right) \phi_1 \right|^2 \\ &+ \left| \left( \partial_\mu - i \frac{g'}{2} y B_\mu - i \frac{g''}{2} 2 Y C_\mu \right) \phi_2 \right|^2 \end{aligned} \quad (14)$$



and

$$\begin{aligned}
 m(W) &= \frac{1}{2^{1/2}} \lambda_1 g \\
 m(Z) &= \frac{m(W)}{\cos \phi} \\
 m(Y) &= \left(\frac{2}{3}\right)^{1/2} \lambda_2 a
 \end{aligned} \tag{15}$$

where

$$\begin{aligned}
 a &\equiv (g'^2 + g''^2)^{1/2} \\
 \cos \phi &= \frac{ag}{b} \\
 b &\equiv (a^2 g^2 + g'^2 g''^2)^{1/2}
 \end{aligned} \tag{16}$$

The effective weak neutral coupling constants of quarks through Z and Y bosons given in equation (6) are obtained by equations (9) and (11):

$$\begin{aligned}
 C_{u_L} = C_{c_L} &= \left(\frac{1}{2} - \frac{2}{3} \sin^2 \phi\right) + \frac{1}{18} \left(\frac{\lambda_1}{\lambda_2}\right)^2 \\
 C_{d_L} = C_{s_L} &= -\left(\frac{1}{2} - \frac{1}{3} \sin^2 \phi\right) + \frac{1}{18} \left(\frac{\lambda_1}{\lambda_2}\right)^2 \\
 C_{u_R} = C_{c_R} &= \left(-\frac{2}{3} \sin^2 \phi\right) - \frac{1}{9} \left(\frac{\lambda_1}{\lambda_2}\right)^2 \\
 C_{d_R} = C_{s_R} &= \left(\frac{1}{3} \sin^2 \phi\right) - \frac{1}{9} \left(\frac{\lambda_1}{\lambda_2}\right)^2
 \end{aligned} \tag{17}$$

These results reproduce the ones for the WS model in the limit of  $(\lambda_1/\lambda_2)^2 \approx \text{small}$ .

With the inputs of

$$\begin{aligned}
 \sin^2 \phi &= 0.22 \\
 \left(\frac{\lambda_1}{\lambda_2}\right)^2 &= 0.435
 \end{aligned} \tag{18}$$

the neutral current coupling constants are compared with experiment in Table II. Our results are as good as the WS model or closer to the mean values. The correction term in equation (17) due to  $Y$  contribution is small. By (15) and (18), we obtain

$$\begin{aligned} m(Z) &= 1.13m(W) \\ m(Y) &= 1.98m(W) \end{aligned} \quad (19)$$

The neutral current coupling of electron through  $Z$  and  $Y$  is given by

$$\begin{aligned} \bar{e}_e = \left( \frac{1}{4} i \right) \frac{g}{\cos \phi} \left\{ + \left[ -(1 - 2 \sin^2 \phi) \bar{e} \gamma_\mu (1 + \gamma_5) e + (2 \sin^2 \phi) \bar{e} \gamma_\mu (1 - \gamma_5) e \right. \right. \\ \left. \left. + \dots \right] Z + \left[ -\frac{2}{3} \frac{a^2}{b} \bar{e} \gamma_\mu (1 + \gamma_5) e + \dots \right] Y \right\} \end{aligned} \quad (20)$$

If we neglect the small  $Y$  contribution, the parity violation (PV) in atomic physics is the same as WS model.

Thus, in the case of the symmetry-breaking  $SU(6) \rightarrow SU(3)^c \times U(1)$ , the effective weak coupling constants are not in good agreement with experiment. But for the symmetry-breaking  $SU(3)^c \times SU(2) \times U(1) \times U(1) \rightarrow SU(3)^c \times U(1)$ , the agreement is closer to the experimental mean values than the WS model.

*Note added in proof.* The recent experiment (Prescott et al., 1978) on parity nonconservation in inelastic scattering of longitudinally polarized electrons from an unpolarized deuterium target is in good agreement with the WS model for  $\sin^2 \phi = 0.02 \pm 0.03$ .

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